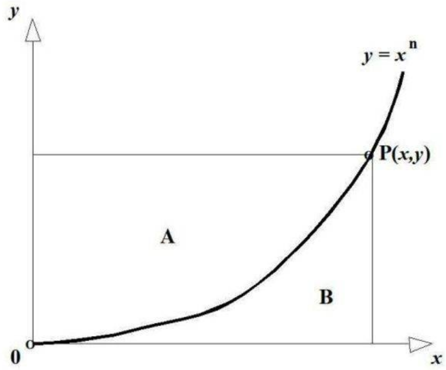
Go straight to the integral power rule

by Sidney Schuman (Lewisham College 1980 - 1995)



Using the mid-ordinate rule, calculate the value of area B for $x = 10$ and n = 2. Check that using 10 vertical strips, $B = 335.05$; $y = 100 \therefore xy = 1000 = \mathrm{A} + \mathrm{B}$. Calculate A & check that $\frac{A}{B} = 2.00256$.

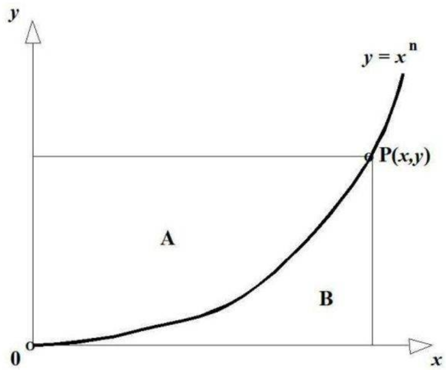
Check that further calculations for n = 3, 4, 5 show that $\frac{A}{B} \approx n$.

Note that the value of $\frac{A}{B}$ is more accurate when more strips are used and with an infinite number of strips, we can say that $\frac{A}{B} = n$.

Putting this together, $A + B = xy = x^{n+1}$ and $\frac{A}{B} = n \therefore A = Bn \therefore Bn + B = x^{n+1} \therefore B(n+1) = x^{n+1} \therefore B = \frac{x^{n+1}}{n+1}$.

The above is a brief look at what can be done by students in the classroom with some encouragement

There is of course a simple proof (below) of the crucial part of this work - the statement that $\frac{A}{B} = n$



Given that $\int x^n dx = \frac{x^{n+1}}{n+1}$, (constant of integration omitted)

then, in the diagram on the left, $B = \frac{x^{n+1}}{n+1}$ Equation 1

Also, $A + B = xy \therefore A + B = x^{n+1} \therefore A = x^{n+1} - B \therefore A = x^{n+1} - \frac{x^{n+1}}{n+1}$

$\therefore A(n+1) = x^{n+1}(n+1) - x^{n+1} \therefore A = n\frac{x^{n+1}}{n+1}$ Equation 2

Comparing equations 1 and 2, we see that $\frac{A}{B} = n$

This is a pre-calculus taster designed as an easy way in for students, many of whom are nervous or downright scared of the topic. Newton's approach, based on fluxions, is not the easiest of concepts for sixth-form students. Area is more familiar to students than gradient, hence the use of the integral power rule as an intrduction.
